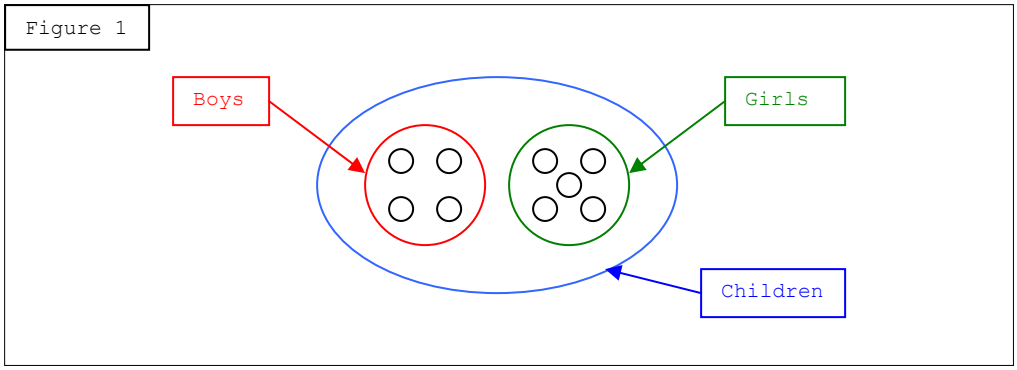
Easy Addition, Easy Subtraction and Algebra

Here is a simple Easy Addition problem.

> Problem 1: There are 4 boys and 5 girls in a classroom. Altogether, how many children are in the classroom?

This example refers to 3 sets of children. There is the set (RED in Figure 1) of 4 boys who are in the classroom. There is the set (GREEN) of 5 girls who are in the classroom. And there is the set (BLUE) of 9 children consisting of the boys and girls together. Of course, this total number of children is unknown until the problem is solved.

Figure 1



Without too much change in the wording of this problem, we can make it into one in which the number of girls (5) is known, and the total number of children (9) is known, but the number of boys needs to be figured out.

> Problem 2: There are SOME boys in a classroom. There are also 5 girls in the classroom. Altogether there are 9 children in the classroom. How many of them are boys?

The words are similar, and Figure 1 is still relevant, but the problem is not the same.

And, of course, we can change the words again to create a problem in which the number of boys (4) is known, and the total number of children (9) is known, but the number of girls needs to be figured out.

Problem 3: There are 4 boys in a classroom. And there are SOME girls in the classroom. Altogether there are 9 children in the classroom. How many of them are girls?

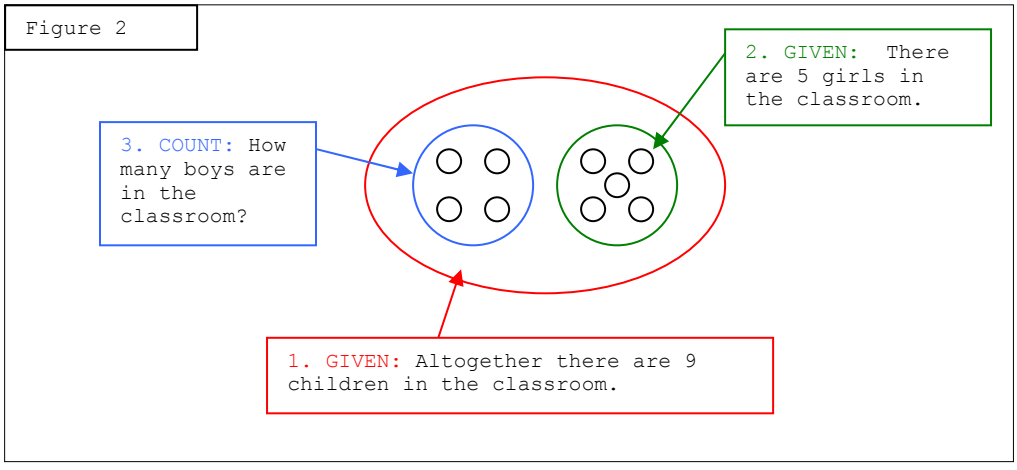
Once again, the words are similar, and Figure 1 is still relevant. But Problems 1, 2, and 3 are quite different from one another.

Problem 1 is an example of Easy Addition. But what type of problem are Examples 2 and 3? And, more importantly, how should children be taught to solve them?

Some children will think that Problems 2 and 3 are also addition problems. They do sound a little like addition problems. So much so that some parents and teachers even mistakenly tell children that the word "altogether" is a "clue" that these are addition problems. In the case of Problem 2 these children will write "5 + 9 = 14." In the case of Problem 3 they will write "4 + 9 = 13."

Problem 2 is in fact an Easy Subtraction problem. But to see it as such will require your child to make a leap in his or her thinking – he or she must be able to build a model using the information that is given in reverse order. This initial step is not an easy one.

Figure 2



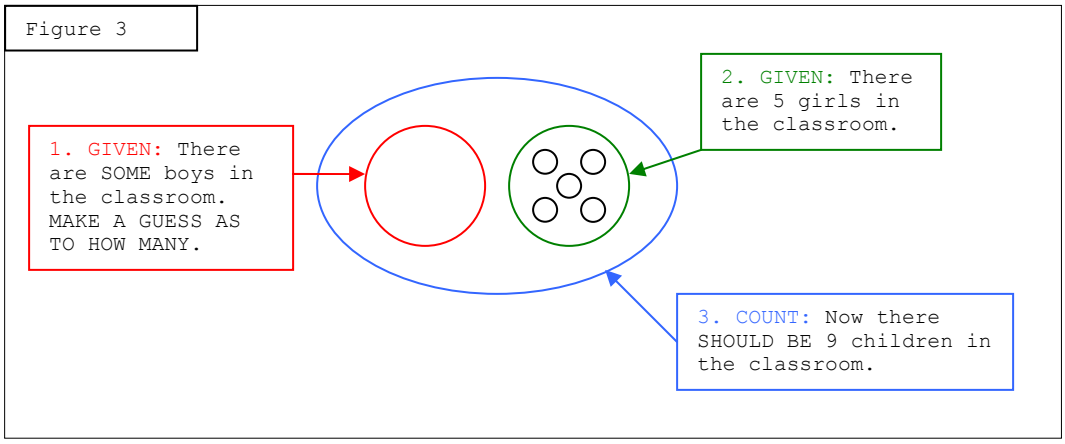
If your child is comfortable thinking in this way you should help him or her to follow these steps.

- (RED in Figure 2) Count out 9 markers to represent all of the children that are in the classroom.
- (GREEN) Separate out 5 of those 9 markers to represent the girls.
- (BLUE) Count the collection of markers that remain – those are the boys who are in the classroom.

Once I am able to re-formulate the problem in this way, it is clear that it really is an Easy Subtraction problem. I can represent it abstractly, in the form of an equation, as 9 – 5 = □.

But there is another way to think about this problem. It requires a different kind of leap in thinking – your child can a) guess at the number of boys and then b) use Easy Addition to check that guess, and then c) amend the guess if necessary. This is also not easy, but solving in this way does use the information given in the problem in the same order in which it is presented.

Figure 3



You can help your child to build a model following these steps:

- (RED in Figure 2) There are SOME boys in the classroom. Encourage your child to make a guess (say 3) and count out that many markers.
- (GREEN) Count out 5 markers to represent the girls in the classroom.
- Move the two sets of markers close together to model the boys and girls together.

- (BLUE) Count the entire collection. (In this case there would be 8, but according to the problem statement there should be 9.)
- (RED) Make a new guess about how many children were originally in the classroom.
- Repeat the process with the new guess, and keep repeating until the guess proves to be correct.

This process of guessing and checking is an excellent beginning to the study of algebra. When I use this approach I am, in effect, solving the equation ☐ + 5 = 9.

The analysis of Problem 3 is just like the analysis above of Problem 2. In this case the number of boys (4) is known as well as the total number of children (9). The number of girls needs to be figured out. Problem 3, like Problem 2 is an example of Easy Subtraction. To see that, your child will need to first think about the total number of children, and then separate out the boys. When thought of in this way, Problem 3 can be represented by the equation 9 – 4 = ☐.

Problem 3 can also be solved algebraically, by first guessing at the number of girls and then using Easy Addition to check. In that case, you are solving the equation ☐ + 5 = 9.

Comments:

a) You should wait until your child has gained some reasonable fluency with Easy Addition and Easy Subtraction before introducing more difficult problems like these. While Problems 2 and 3 can both be solved by using either Easy Addition or Easy Subtraction, both routes require some additional thinking.

b) In general I emphasize the need to explicitly show children how to model the various kinds of word problems. The problems introduced here are different. Children who know how to solve Easy Addition and Easy Subtraction problems can, and should, learn to solve problems like those introduced above, largely on their own. The solutions can be derived logically from what they already know.

c) It is because children can learn to solve these kinds of problems largely on their own that when you do introduce them, you should be sensitive to the route that your child is trying to follow. Let him or her make the first move in building the model. Support the path that your child takes.

d) On the other hand, it is important that children learn to think about problems like these in both ways. At some point you should encourage both methods of solution. The trick is to avoid requiring your child to use methods that he or she really doesn't understand.

e) Problems 2 and 3 are examples of Easy Subtraction that sound like they might be examples of Easy Addition. It is also possible to make up word problems that are examples of Easy Addition but sound like they might be examples of Easy Subtraction. Here is an example.

> Trixie had some cupcakes. After Punch, the dog, ate 3 of them Trixie had 2 cupcakes left. How many cupcakes did Trixie start with?

Your child can solve this problem by first thinking about the number of cupcakes that Trixie has left (2) and then thinking about the number of cupcakes that Punch ate (3). Thought about in this way the problem is clearly and example of Easy Addition that would be represented abstractly by the equation 2 + 3 = □. But one might also solve this problem algebraically, using the Easy Subtraction model, by first guessing at the number of cupcakes that Trixie started with, taking 3 of those cupcakes away, and counting to see if 2 cupcakes are left. In that case, the problem would be represented abstractly by □ – 3 = 2.